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Evaluating the Possibility of Asteroid Rock Conctituents Dispersion

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Abstract

In interception large asteroids at small distances from the Earth, when the time prior to impact is several hours or less, very high explosions yields are required to destruct asteroid into fragments posing no hazard to ecology. Under these conditions of great importance is the increase of the factor converting the explosion energy into kinetic energy of scattering fragments, which can be achieved by double sateroid affecting. The first "weak" effect makes sateroid fragments disperse at the velocity no more than the escape velocity (relative to asteroid). While asteroid is under dispersed condition, a more powerful charge is introduced into its center, which comes into action with collapse of asteroid rock constituents and provides a high factor of explosion energy transition to kinetic energy of its fragmets.

The efficiency of this method of asteroid affecting is the objective of present paper.

1. We consider conditions of asteroid rock constituents dispersion, when asteroid is first affected by a low-power explosion .

It is assumed that mechanically affected asteroid rock constituents uniformly disperse in space. Asteroid has a spherical shape. The fragment motion on the outer asteroid surface is described by the set of equations

$$\frac{dR}{dt} = v , \tag{1}$$

$$\frac{dv}{dt} = -\frac{GM}{R^2} , \tag{2}$$

with $t = 0$, $R = R_0$, $v = v_0$, initial conditions, where R is the asteroid radius, M is the asteroid mass, v is the velocity of its surface motion, $G = 6{,}7.10^{-8} \text{cm}^3\text{g}^{-1}\text{s}^{-2}$ is the gravity constant.

Having divided the right and left sides of the set of equations (1), (2) termwise, we obtain

$$\frac{dR}{dv} = -\frac{vR^2}{GM}$$

equation, whose solution has the following form:

$$GM\left(\frac{1}{R_0} - \frac{1}{R}\right) = \frac{v_0^2 - v^2}{2} . \tag{3}$$

It specifies the maximum asteroid expansion dependig on its initial surface velocity v_0:

$$1 - \delta^{\frac{1}{3}} = \frac{1}{\beta} , \tag{4}$$

where $\delta = \dfrac{\rho}{\rho_0} = \left(\dfrac{R_0}{R}\right)^3$ is the ratio between the density at the end of the scattering stage and initial asteroid density,

$\beta = \left(\dfrac{V}{v_0}\right)^2$, $V = \sqrt{\dfrac{2GM}{R_0}}$ is the escape velocity (relative to asteroid).

From (2) and (3) we derive the time relationship:

$$\bullet \qquad t = -GM\int_{v_0}^{v}\left(\frac{GM}{R_0} - \frac{v_0^2 - v^2}{2}\right)^{-2} dv. \qquad (5)$$

•The total time of asteroid being in an expanded stage
$T=2t_{v=0}$ can be given as

$$\bullet \qquad T = 2\frac{R_0}{V}\frac{1}{\sqrt{\beta}}I, \qquad (6)$$

where

$$I = \int_0^1\left(1 - \frac{1-x}{\beta}\right)^{-2}\frac{dx}{\sqrt{x}} = \frac{\beta}{\beta-1}\left(1 + \frac{\beta}{\sqrt{\beta-1}}arctg\frac{1}{\sqrt{\beta-1}}\right) \qquad (7)$$

(it has been obtained from (5) by substituting $v = v_0\sqrt{x}$).

Relations (4), (6) и (7) specify the time of asteroid being in an expanded state T depending on the required dispersion of asteroid rock constituents $\delta = \frac{\rho}{\rho_0}$. It should be noted that the dimension factor in relation (6)

$$\frac{R_0}{V} = \sqrt{\frac{R_0^3}{2GM}} = \frac{0,133.10^4}{\sqrt{\rho_0}}\quad s \qquad (8)$$

(ρ_0 is in g/cm^3) depends solely upon the initial asteroid densiny and is undependent of its other parameters (dimesions, mass). By this is meant that the derived relationship is also true for all fragments within the asteroid space, i. e. all parts of asteroid move during scattering and later compression in a similar manner. In particular, the state corresponding to the escape velocity is attained for all asteroid fragments at one time. It is not unlikely that this situation is trivial.
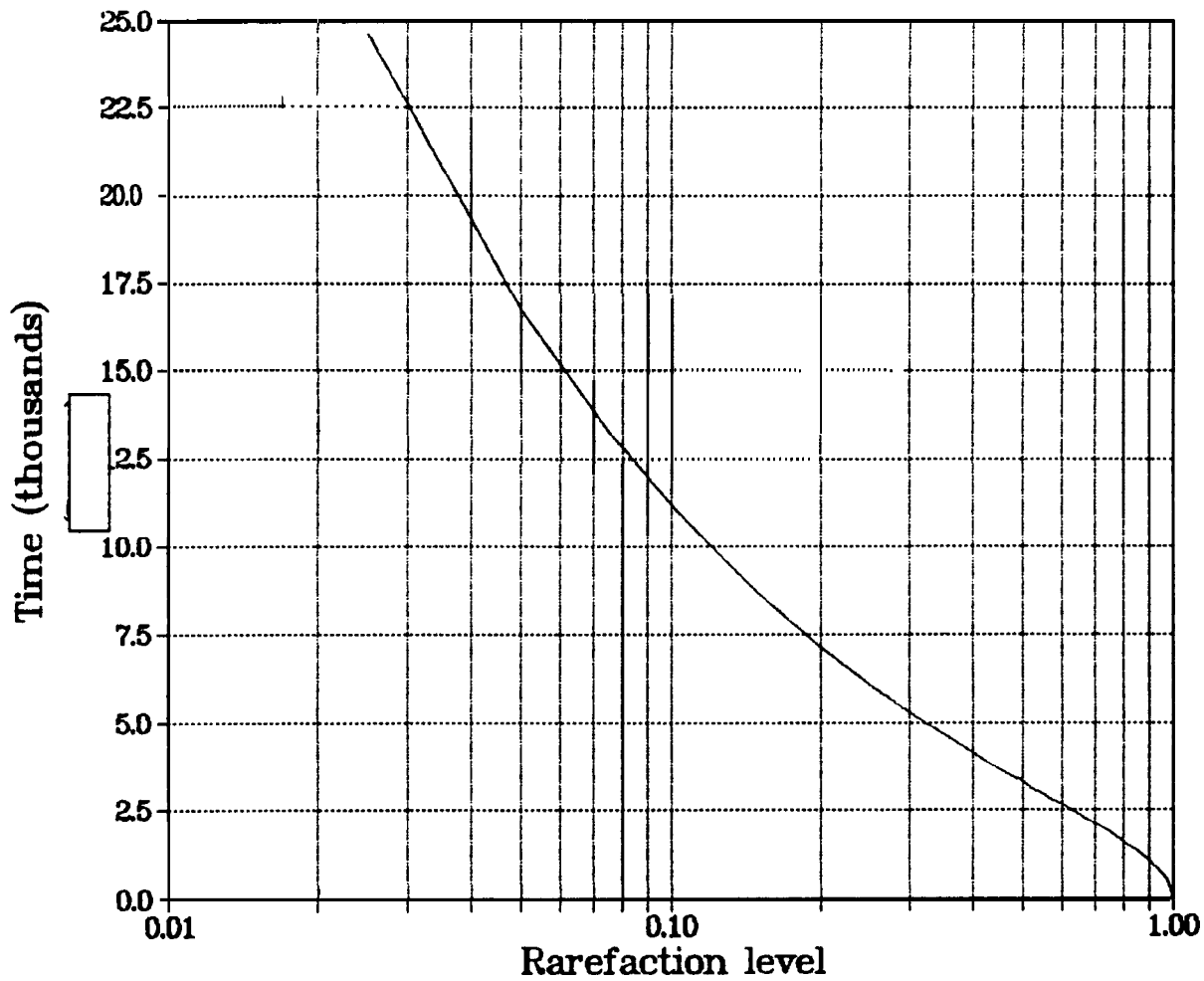
Fig. 1.

Dependance of time of asteroid being under dispersed conditions on its rarefaction level, which is described by relations (4), (6), (7) and (8).

$\delta = \dfrac{\rho}{\rho_0}$ has been plotted on the abscissa, $T\sqrt{\rho_0}$ has been laid off as ordinate (T is in seconds, ρ_0 is in g/cm^3).

By way of example let us find, how much time the asteroid, having the density of rock constituents $\rho_0 = 3$ g/cm^3, will be in an expanded state, if it is required to disperse it in an average

density by an order of magnitude $(\delta = 0,1)$. In this case we derive $T=0,64.10^4$s, i. e. about two hours irrespective of asteroid dimentions or its mass..

2. We estimate the kinetic energy W that should be spent on asteroid expansion to the required averige density. As the asteroid radius increases f rom R_0 to R the following energy is spent on expanding the spherical layer having initial dimensions r_0, $r_0 + dr_0$:

$$dW = \frac{Gmdm}{r_0}\left(1 - \frac{R_0}{R}\right), \quad m = M\left(\frac{r_0}{R_0}\right)^3, \quad dm = 4\pi r_0^2 \rho_0 dr_0, \quad (9)$$

where m is the mass of asteroid part within the spherical layer. Integrating (9) with respect to the asteroid space, we derive

$$W = \frac{3}{5}\frac{M^2 G}{R_0}\left(1 - \delta^{\frac{1}{3}}\right). \quad (10)$$

Consider an example: the radius of asteroid is $R_0 = 100$ m, the density $\rho_0 = 3$g/cm³, the required dispersion of rock constituents is $\delta = \frac{\rho}{\rho_0} = 0,1$. Based on relation (10), the kinetic energy needed for that will be $W = 3.4 \cdot 10^{14}$erg$=8,1$ kg of explosive. If ~1% of explosion energy goes over into kinetic energy of scattering, the energy of ~1t of explosive will be needed for the required dispersion of asteroid rock constituents (by an order of magnitude in an average density).

It should be noted that in the case above it will take only twice as much energy $(6,35.10^{1\;4}$erg) to give the escape velocity to asteroid fragments (in relation (10) $\delta=0$). That is in our example the asteroid scattering with no later gathering of its fragments can be easily done without the second effect produced by explosion of a more powerful charge at its centrer. The efficiency of double

effecting would increase, if the level of rock constituents rarefaction δ lowered. However, in this case the delivery of the second charge to the asteroid center poses greater difficulties. .

It seems likely that the low power of the first ("weak") effect W as compared with the power nesessary for imparting the escape velocity to asteroid fragments $W_2 = \dfrac{3}{5}\dfrac{M^2 G}{R_0}$ could be taken as the criterion for the double affecting efficiency, that is

$$\frac{W}{W_2} = \left(1 - \delta^{\frac{11}{13}}\right) << 1. \tag{11}$$

Non-fulfilment of this condition means that the power of the first ("weak") effect is comparable to the power necessary for imparting the escape velocity to asteroid fragments, L e. the problem of irrevocable asteroid dispersion can be solved without the second explosion, for which purpose the first effect energy release is to be slightly increased (as in the above example). On the other hand, satisfaction of requirement (11) limits the possibility of asteroid rock constituents dispersion in an average density by $\delta^{\frac{1}{3}} \sim 1$ region, that is the possibility of the second more powerful charge delivery to the central asteroid region. Note that the conclusion made is independent of particular asteroid parameters.

The interrelation between $\dfrac{W}{W_2}$ and $\delta = \dfrac{\rho}{\rho_0}$ which is specified by (11), is shown in Fig. 2 as an illustrative example. .
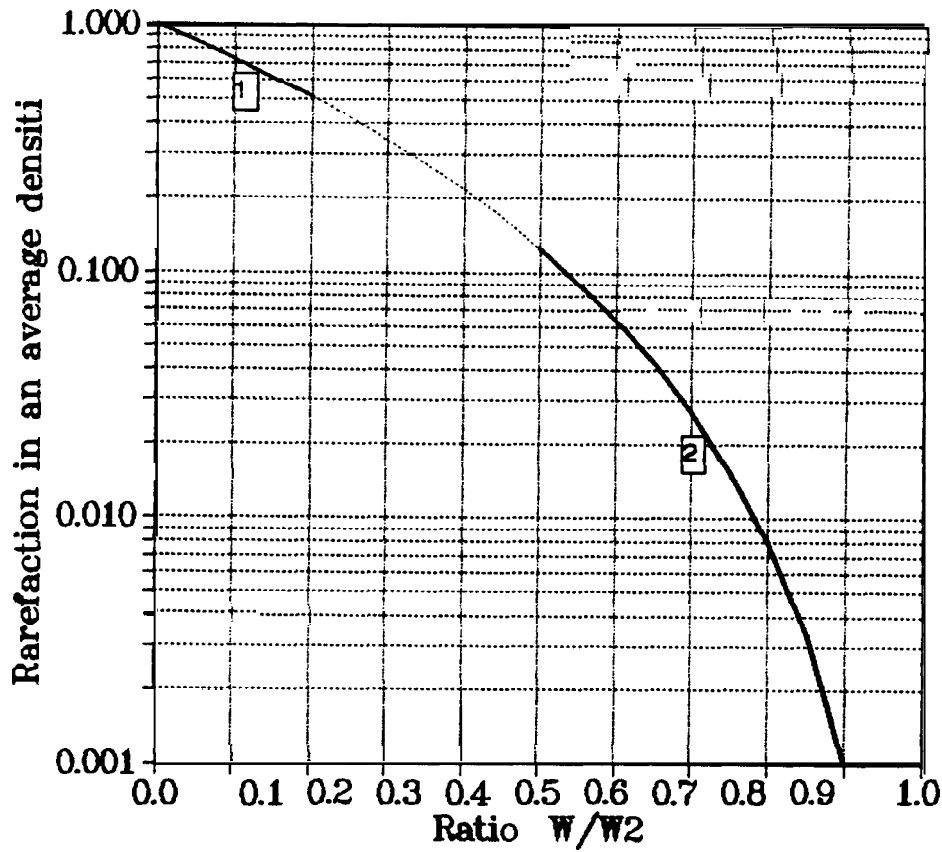
Fig. 2

Interrelation between the power of the first effect W produced upon asteroid and dispersion in an average rock constituents density $\delta = \dfrac{\rho}{\rho_0}$, which is attained in this case.

Portion [1] in Fig. 2 corresponds to rather low values of the first explosion energy release W as compared to W_2, therefore the tactics of double asteroid affecting can hardly be considered as justified $(W \leq 0{,}2W_2)$. Portion [2] corresponds to the condition, according to which asteroid rock constituents dispersion is to be sufficient for making it possible to implement thesecond more powerful effect $(\rho \leq 0{,}1\rho_0)$. We see that these regions do not meet, which points to the difficulties associated with implementation of efficient double asteroid affecting irrespective of its parameters.

3. The above considerations are true for the case of "distant interception", when asteroid is affected in advance and there are no time limitations. However, in "near interception" under limited time conditions the escape velocity imparting to asteroid fragments may turn out to be insufficient for the required asteroid rock constituents dispersion prior to its impact against the Earth. To estimate the asteroid expansion velocity in this case we refer to equations (1) and (2), having added $t = \infty$, $R = \infty$, $v = 0$ condition to them. We have

$$v = \left(\frac{2GM}{R}\right)^{\frac{1}{2}} . \qquad (12)$$

Note that the relations close to those above can be also derived for other considered asteroid expansion models (breaking down into two or more fragments).

Substituting (12) in (1) we obtain the time dependence of asteroid radius R

$$\frac{2}{3}\left(R^{\frac{3}{2}} - R_{0}^{\frac{3}{2}}\right) = \sqrt{2GM} . t , \qquad (13)$$

or

$$\delta = \frac{1}{\left(1 + \sqrt{6\pi G \rho_0} \, t\right)^2} . \qquad (14)$$

Our interest is with the late scattering snage, when $\delta \ll 1$. In this case relatione (14) takes the following form:

$$\delta = \frac{1}{6\pi G \rho_0 t^2} = \frac{0,79.10^6}{\rho_0 t^2} , \qquad (15)$$

where the time prior to impact against the Earth t is expressed in secondes, the initial asteroid density ρ_0 is in g/cm^3. For example, if there is a day before impacting against asteroid,

whose rock constituents density is 3 g/cm^3, asteroid having been affected with imparting the escape velocity to its fragments will disperse in an average density by a factor of $\delta^{-1} \approx 2,8 \cdot 10^4$ before falling on the Earth. If the sateroid radius is R_0 =100m, its fragments will disperse over the area with radius $R = R_0 \delta^{-\frac{1}{3}} \approx 3$ km, which is known to be insufficient for preventing after-effect of its fall. Thus, to disperse asteroid over rather large areas in "near interception" releases of energy are required, which impart velocities to its fragments being much higher than the escape velocity, $v_0 >> V$. Dependence of asteroid dimention on its scattering time defined by relation (3) and (5) can be presented for this case as

$$\tau = \frac{V}{R_0} t = \frac{\beta^{\frac{3}{2}}}{1-\beta} \left\{ \frac{\sqrt{x}}{x+\beta-1} - \frac{1}{\beta} + \frac{1}{\sqrt{1-\beta}} \ln \left[\frac{\left(\sqrt{1-\beta}+1\right)}{\left(\sqrt{1-\beta}+\sqrt{x}\right)} \sqrt{\frac{x+\beta-1}{\beta}} \right] \right\}, \quad (16)$$

where $\qquad x = 1 - \beta \left(1 - \frac{R_0}{R} \right)$.

Recall designations: $\qquad x = \left(\frac{v}{v_0} \right)^2$, $\qquad \beta = \left(\frac{V}{v_0} \right)^2$.

In the limiting case of $\beta \to 1$ expression (16) goes into (15). We are interested in the limiting case of "rapid expansion", when $\beta \to 0$. In this case formula (16) becomes an evident relation

$$R = R_0 + v_0 t \approx v_0 t,$$

or expressing the initial asteroid surface velocity v_0 in terms of the total kinetic ehergy of asteroid fragments relative to its center during a uniform expansion $W = \frac{3}{2} \frac{M v_0^2}{R_0^5} \int_0^{R_0} r_0^4 dr_0 = \frac{3}{10} M v_0^2$, we find that to expand asteroid to R radius the following kinetic energy should be imparted to its fragments:

$$W = \frac{3}{10} M \left(\frac{R}{t} \right)^2 .$$ (17)

We take the falling asteroid energy E dispersion over area $\alpha = \frac{E}{\pi R^2} = 10^{15}$ erg/m^2 as ecologically safe. This value is comparable to thermal solar radiation energy release over a unit of the Earth surface area during twenty-four hours. For the asteroid having rtadius $R_0 = 100$ m and rock constituents density $\rho_0 = 3$g/cm^3, which is moving to the Earth at the velocity of 25km/s, and has kinetic energy 10^3 Mt of TNT equivalent, such density of energy release can be realized with the radius of the falling area of its fragments $R \cong 100$ km. Relation (17) specifies the kinetic energy needed for that, which should be imparted to asteroid fragments, depending on the interception time t (from explosion to potential fall on the Earth), $W \cong \frac{3,8.10^{26}}{t^2}$ erg, where t is expressed in seconds. If asteroid interception has been made at a distance of 10^5 km from the Earth, L e. an hour before its fall, the required kinetic energy imparted to fragments will be $W \cong 0,3.10^{20}$erg $\cong 0,7$ kt of TNT equivalent. If ~1% of explosion energy goes over into kinetic energy of scattering, the energy of ~100 kt of TNT equivalent is needed for the required dispersion of asteroid rock constituents..

For illustration purposes Fig. 3 shows W dependence on the interception time for asteroid having $R_0 = 100$m and 1000m.
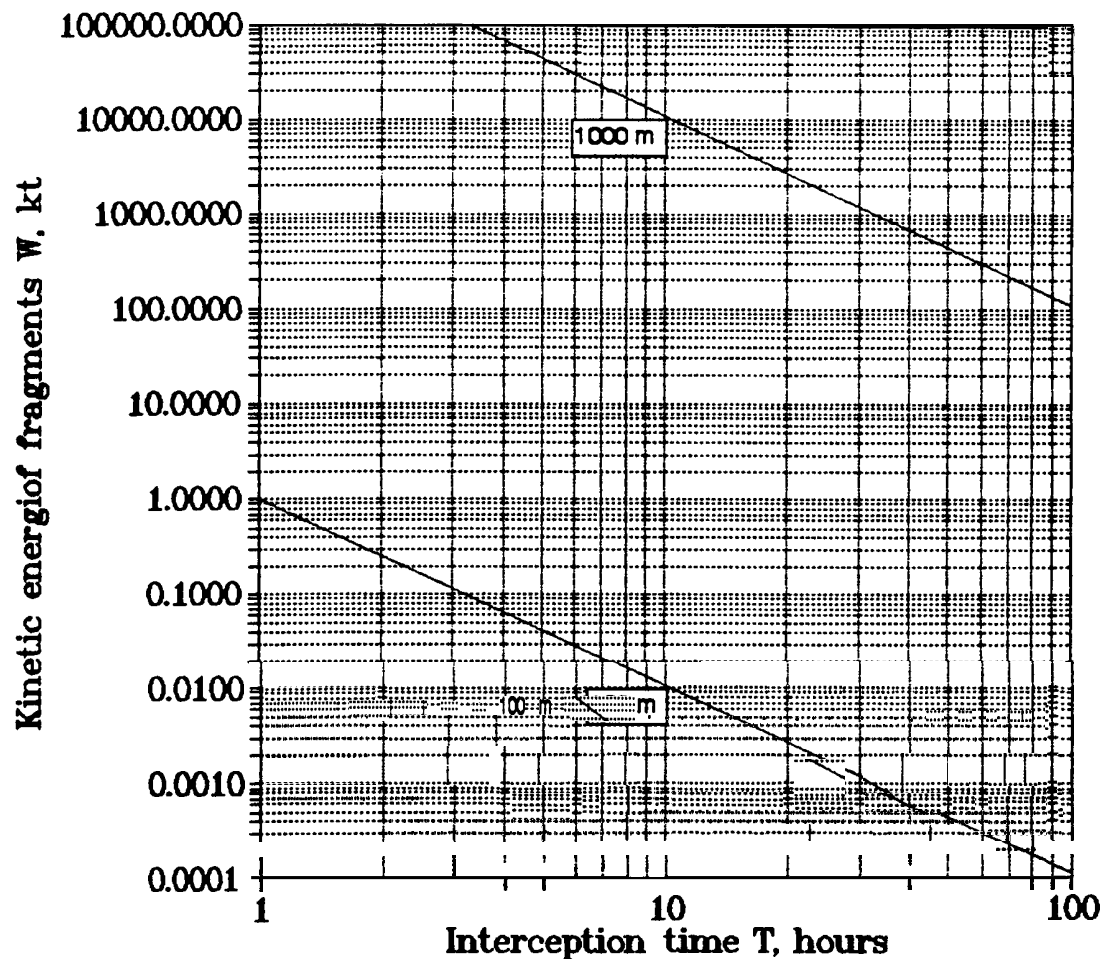
Fig.3

Dependence of kinetic energy of asteroid fragments, necessary for their dispersion to the required level, on the interception time (before its fall on the Earth) for asteroids having $R_0 = 100$ m and 1000 m.

As Fig. 3 shows, the energy release necessary for asteroid neutralization rises sharply as its dimention increase $(W \sim R_0^5)$. Therefore, to intercept large asteroids having $R_0 > 100$m of great importance is the increase of the factor converting the explosion energy into kinetic energy of fragments, which can be attained through double asteroid affecting: the first "weak" effect is used to disperse fragments; it is followed by a more powerful explosion at the center after asteroid rock constituents have collapsed.